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Language and Modeling Word Problems in Mathematics Among Bilinguals

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ABSTRACT. The study was conducted to determine whether the language of math word problems would affect how Filipino–English bilingual problem solvers would model the structure of these word problems. Modeling the problem structure was studied using the problem-completion paradigm, which involves presenting problems without the question. The paradigm assumes that problem solvers can infer the appropriate question of a word problem if they correctly grasp its problem structure. Arithmetic word problems in Filipino and English were given to bilingual students, some of whom had Filipino as a first language and others who had English as a first language. The problem-completion data and solution data showed similar results. The language of the problem had no effect on problem-structure modeling. The results were discussed in relation to a more circumscribed view about the role of language in word problem solving among bilinguals. In particular, the results of the present study showed that linguistic factors do not affect the more mathematically abstract components of word problem solving, although they may affect the other components such as those related to reading comprehension and understanding.

Key words: bilinguals, language, math problem solving, word problems

WHAT IS THE EFFECT ON PROBLEM SOLVING of using a bilingual's first or second language in semantically rich domains such as school mathematics? The question is both of theoretical and practical interest. Cognitive psychologists seek to understand how information is represented, organized, and processed by the human mind as it engages in different types of tasks. There is particular interest in how the mind processes pieces of information that are equivalent conceptually but are dissimilar in form. This interest is reflected in the body of research on cognitive processes such as analogical problem solving, case-based reasoning, and comprehension of metaphor and simile, because such processes reveal how the human mind represents information about different types of experiences. In this regard, cognitive psychologists have become increasingly interested in a bilingual person's ability to process concepts using two distinct sets of structures and processes of language.

Many questions have been raised regarding how a bilingual's knowledge is represented in two languages, but the questions often relate to whether the bilingual's two languages access one common conceptual system or whether each language accesses a different conceptual system. Much of the research that seeks to address this question relates to the bilingual's lexicon and memory for words and concepts. Recent reviews of the empirical data by Francis (1999) and by French and Jacquet (2004) suggest that much of the research supports the view that words in one language and its translations in another language share the same conceptual representation in the mind of the bilingual.

However, there are some variations in the results that may lend themselves to alternative interpretations or theoretical explanations. For example, studies by Paivio, Clark, and Lambert (1988) suggest that the shared conceptual representation of translation equivalents might be different for concrete and abstract words. Paivio's (1991) theoretical explanation proposes that concrete words from two languages have more shared conceptual elements than do abstract words because concrete words have common referent images that abstract words do not. Magiste's (1985) research also suggests that there might be developmental or age-related differences in how conceptual representations are established. However, it should be underscored that these results do not contradict the basic notion that there is some shared conceptual representation of words in the bilingual's memory. Indeed, these variations in the results merely point to finer theoretical and possibly methodological issues related to the study of bilingual memory for words.

Although these theoretical issues relate to memory and the lexicon, they have important implications and applications to formal education in bilingual and multilingual communities. Formal education is concerned with helping students develop knowledge and skills in a wide variety of domains. In bilingual communities, students are required to learn and process information that is presented in their first and second languages. Questions are often raised regarding whether bilingual students' learning is affected by the language used in instruction. Is knowledge acquired by bilingual students dependent on, or independent of, the language systems used to teach and learn the pertinent concepts and skills? In an increasingly global environment, more and more students are becoming bilingual by choice or by force of circumstance. As such, these ques-

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tions are relevant to an increasing number of students in a growing number of schools and educational systems.

In the present study, I address the matter of the relationship among learning, knowledge, and language by investigating the effects of using Filipino–English bilingual students' first and second language in solving word problems in arithmetic. Word problems were chosen as the focus of the study for two important reasons. First, word problems have always been an important part of mathematics education. Verschaffel, Greer, and De Corte (2000) reported that word problems can be found in 4,000-year-old Egyptian papyri, in Greek and Roman manuscripts, and in the earliest printed arithmetic textbooks. Currently, efforts to measure mathematics achievement across different countries rely heavily on word problems. Second, word problems have a clear linguistic component because the problem elements are embedded within a text, but the quantitative elements of the problem can be conceived in abstract symbolic but nonlinguistic forms.

Language and Math Word Problems

A number of researchers have found effects of variations in the linguistic aspect of word problems on problem representation and problem-solving performance. Cummins, Kintsch, Reusser, and Weimer (1988) and Riley and Greeno (1988) showed that the difficulties in understanding certain types of word problem texts led to more errors in the solution for those types of problems. Teubal and Nesher (1991) also reported that the lack of correspondence between the order of mention of events and the order of numerical data referring to the different events in the problem texts affected solution performance among problem solvers. De Corte, Verschaffel, and De Win (1985) and De Corte and Verschaffel (1987) also found that rewording the text of word problems so that they more closely reflect the problem structure led to significant improvements in problem-solving accuracy. All these studies underscore the effects of language-related factors on understanding and solving word problems.

There have been some related studies done with bilingual students solving word problems in their first and second languages. For example, Clarkson (1991) reported on the problem-solving performance of Papua, New Guinea, students when tested using word problems in English, which was not their first language. Clarkson found that 33% to 39% of the students' errors were language related (i.e., reading and comprehension errors). He also reported that such errors occurred at lower levels among native English speakers, and on that basis suggested that the difficulties of those students were partly owing to their lack of proficiency in English.

Bernardo (1999) also found that Filipino–English bilingual students were better at solving typical word problems in arithmetic when the problems were written in their first language (Filipino) rather than in English. Error analysis suggested that better comprehension of the text was the reason for the improved performance on first-language problems. In the same study, Bernardo found that rewording the problem texts in the same manner as that in the De Corte et al. (1985) study resulted in larger gains in accuracy with the first-language problems.

More recently, Bernardo (2002) used a recall paradigm to more directly assess Filipino–English bilingual students' understanding of word problems in Filipino and in English. The study revealed a first-language advantage; students were better able to understand and solve the problems in their first language and had more difficulties understanding the same problems in their second language (see also Bernardo & Calleja, 2005). These findings suggest that some difficulties that students have in relation to understanding word problems might be intensified in the case of bilingual students who have to solve word problems written in their second language.

However, it is not always the case that language affects word problem solving performance among bilingual students. For example, data from studies cited earlier (Bernardo, 1999; Riley & Greeno, 1988; Teubal & Nesher, 1991) suggest that older students are less likely to experience difficulties associated with language-related aspects of word problem solving. These results were attributed to the students' acquisition of problem schemas in the domain. Problem schemas are abstract representations of the underlying structure of the word problems that allow problem solvers to model and solve the word problems in ways that are largely unaffected by the superficial (e.g., linguistic) features of the problems (Bernardo, 2001).

Some other studies with Filipino–English bilinguals suggest a less significant role of language in word problem-solving performance. For example, Bernardo (1996) found no evidence that the language of the problem affected the tendency to interpret subtechnical terms such as "more" and "less" in specialized ways that are specific to the mathematics problem-solving context. Bernardo and Calleja (2005) also found no evidence that the language of the problem affected the tendency to ignore real-life considerations in modeling word problems in mathematics.

Both of these results can be explained by the use of abstract problem schemas. Reyes (2000) studied students solving statistics problems and found that testing the students in their first language facilitated the access and use of conceptual knowledge; however, these benefits were not found when students had to access computational knowledge. Bernardo's (1998) studies on analogical problem solving among Filipino–English bilinguals showed that overall analogical transfer was better when the language of the source and target problems was the same. But his results also suggest that this language-compatibility effect seemed to reside mainly in the process of retrieving the source problem; the results also suggest that language makes no difference in the actual process of applying the source information to the target problem. These results show that language factors do not seem to affect the more mathematically abstract components of word problem solving.

The various studies just cited suggest that the effect of language factors on problem solving in mathematics probably depends on the level of mathematical abstractness of the specific component processes involved. Various theorists of word problem solving (Carpenter, Moser, & Bebout, 1988; Kintsch & Greeno, 1985) have proposed that solving a word problem involves a complex and interrelated set of information-processing components. These components may vary in their level of mathematical abstractness or the degree to which the processes involve the manipulation or transformation of mathematical concepts into mathematical symbolic representations. Some of these components are the basic linguistic processes involved in comprehending text and the basic memory retrieval processes and may thus be considered relatively low in mathematical abstractness. (The linguistic comprehension of texts also involves the processing of abstract linguistic symbolic representations, but these processes are not mathematically abstract because the concepts involved are not represented in mathematical symbolic forms.) However, some components involve the direct transformation or manipulation of information that is represented and organized in mathematical symbolic forms. These refer to processes such as the application of quantitative operations, the construction of models of the quantitative information, the comparison of such models, and the transfer of problem solutions.

In the present study, I inquired into the role of language in another mathematically abstract component of word problem solving: modeling the problem structure. Various theories of word problem solving (Carpenter et al., 1988; Kintsch & Greeno, 1985) assume that one of the most important processes in solving word problems is the construction of the abstract problem structure based on the textual proposition of the problem. Thus, beyond simply reading the text of the word problem for linguistic comprehension, students need to model the implicit problem structure that specifies the known and unknown elements of the problem and how these are related to each other. A student then chooses and executes the solution procedures based on the problem structure.

One of the ways by which researchers have studied this process of modeling the problem structure is by using the problem-completion paradigm (Cummins et al., 1988; Krutetskii, 1976). The paradigm involves presenting word problems with the question omitted and then asking the students to provide the correct question. The procedure assumes that the students can logically infer the question from the given problem information if they grasp the quantitative relationships that underlie the problem structure. In other words, if the students correctly understand the problem structure, they should be able to infer the question that correctly follows from the given information.

In this study, Filipino–English bilingual students were presented arithmetic word problems in Filipino and in English. The questions were omitted in all problems and the students were first asked to complete the problem by supplying the correct question and then solve the problem. Two groups of bilingual students participated in the study: one group had Filipino as its first language, and the other group had English as its first language. The word problems were classified as easy or difficult based on a previous analysis of the problems. I hypothesized that the mathematically abstract process of modeling the problem structure would not be affected by the language of the problem; students would be equally successful in completing and in solving the word problems, regardless of the level of difficulty of the problems and the first language of the students.

Method

Participants

The participants were 111 fourth graders from 10 private elementary schools that cater to students from families in the middle to upper income brackets. All the students reported that they spoke, read, and understood Filipino and English. Of the total, 57 students reported that Filipino was the language they used at home, and 54 reported that English was the language used in their homes. The reports were verified by interviewing the homeroom teachers of the students about the language behaviors of the children. All the students were being instructed in mathematics using the English language, and English-language textbooks, learning materials, and assessment instruments.

Materials

The problems used in the study were based on the 18 story problems originally used by Riley and Greeno (1988) and were similar to those used by Bernardo (2002). However, the question was omitted in each of the problems, as shown in the Appendix.

The 18 story problems were composed of six specific problems within each of the three major problem types. The problem types were as follows:

1. *combine* problems, in which a subset or superset must be computed given information about two other sets;
2. *change* problems, in which a starting set is changed by transferring items in or out, and the number of the starting set, transfer set, or the results set must be computed given information about two of the sets; and
3. *compare* problems, in which the number of one set must be computed by comparing the information given about sets.

All the word problems included only the numbers 1 to 9, and the correct answers ranged only from 1 to 10. All contained Judy and Carlo as actors and candies as objects. This procedure was done to reduce the memory load requirements of the task; that is, the students only had to attend to the numbers stated in the problem and their described relationships.

The 18 word problems were first written in English and then translated into Filipino by a research assistant who was fluent in both Filipino and English. To ensure that the translations were equivalent, another research assistant translated the Filipino translations back into English.

The set of 18 English problems and the set of 18 Filipino problems were arranged in four different random sequences. Each problem was written using big print on 5 × 8-in. cards. Each set of 18 problems in one random sequence was combined to make a small booklet. Each student worked on 36 problems, 18 in Filipino and 18 in English. For each student, a different sequence was used for the Filipino problems and for the English problems. About half of the students were randomly chosen to work on the English language problems first; the rest worked on the Filipino language problems first.

Based on earlier research (Cummins et al., 1988) that used the same types of word problems, 6 problems were identified as easy problems, and the other 12 were difficult problems.

Procedure

The students were tested individually in a reasonably quiet place in their school during school hours. All the problems were presented to the students by showing the cards with the written problems. While the student was reading each problem, the experimenter also read the problem aloud twice. The student was first asked to complete the problem by providing a question that would be appropriate, given the first part of the problem. The student was then asked to solve the problem that he or she determined. The student's responses were tape recorded. All students were told that only the researchers would hear the tapes, and no student objected to the session being recorded.

Results

Each of the students' responses was coded as showing correct problem completion or not. Correct problem completion was scored if the student generated a question that was identical to or similar in content to the omitted question. Thus, a correct problem always inquired about the same unknown quantity as the original question. All other responses were coded as incorrect. The proportions of correct problem completions for the two types of problems for the two groups of students are summarized in Table 1.

I analyzed the data using a 2 × 2 × 2 analysis of variance (ANOVA) for mixed factorial designs, with the first language of the student as a between-groups variable, and the language of problem and problem difficulty as within-group variables. As expected, the analysis indicated a main effect of problem difficulty, $F(1, 109) = 70.57$, $MSE = .13$, $p < .0001$; the students correctly completed more of the easy problems (M = 87.4%) than difficult ones (M = 69.9%).

With regard to the main research problem, there was no main effect of language of the problem, $F(1, 109) = 2.57$, $MSE = .02$, $p > .10$. This result suggests that the students were equally successful in completing the Filipino and the English problems. Moreover, there was no statistically significant interaction

TABLE 1. Percentage and Standard Error of Correct Problem Completion

	First language			
	Filipino		English	
Problem	*M*	*SE*	*M*	*SE*
Filipino				
Easy	82.5	3.1	92.6	1.6
Difficult	58.6	4.4	86.2	2.4
English				
Easy	82.5	2.7	92.5	1.6
Difficult	56.2	4.5	79.9	2.7

TABLE 2. Percentage and Standard Error of Correct Problem Solution

	First language			
	Filipino		English	
Problem	*M*	*SE*	*M*	*SE*
Filipino				
Easy	84.8	3.2	92.0	1.7
Difficult	61.7	3.9	85.6	2.0
English				
Easy	84.5	2.7	90.7	2.0
Difficult	64.3	4.1	80.4	2.7

between the first language of the student and the language of the problem, and there was no three-way interaction among the three variables in the study (both *F*s < 1.0). That is, both those students whose first language was Filipino and those whose first language was English were equally successful in completing the Filipino and the English problems, and the pattern was also the same with the easy and the difficult problems.

However, there was an unpredicted main effect of the student's first language, $F(1, 109) = 26.81$, $MSE = .13$, $p < .0001$. It seems that students whose first language was English were more successful at completing the difficult problems than were those whose first language was Filipino (*M*s were 86.2% and 65.8%, respectively).

The students' solutions were also coded for accuracy. The proportions of correct solutions for each experimental condition are summarized in Table 2. I analyzed the data using a 2 × 2 × 2 ANOVA, and the results paralleled those found in the problem-completion data. There was also a significant main effect of problem difficulty, $F(1, 109) = 74.21$, $MSE = .03$, $p < .0001$. Consistent with the problem-completion results, there was also no main effect of the language of the problem, $F < 1.0$, no interaction between the first language of the student and the language of the problem, $F(1, 109) = 2.17$, $MSE = .02$, $p > .10$, and no three-way interaction among the three variables, $F(1, 109) = 2.27$, $MSE = .01$, $p > .10$. I also found the unexpected main effect of the first language of the student, $F(1, 109) = 16.41$, $MSE = .12$, $p < .0001$.

Discussion

In this study, I hoped to answer the question: Does the language of the problem affect bilingual students' ability to model the structure of word problems in arithmetic? The results of the study indicate that the answer is no. Students were equally successful in modeling the structure of problems in Filipino and in English, whether their first language was Filipino or English. The students were just as likely to succeed with the easy problems in both languages and equally likely to have more trouble with the difficult problems in both languages.

The results of the study are consistent with previous research that demonstrates the independence of the more mathematically abstract components of word problem solving from the influence of language factors. Such processes, including the process of modeling the problem structure of word problems, operate using mathematical knowledge and procedures that are represented differently from, for example, more linguistic, visual, or kinesthetic information. The use of such knowledge may be driven by problem schemas or abstract representations that are no longer significantly affected by superficial elements of the problem, such as the language of the problem. Indeed, the students who participated in the study were already in their fourth grade of formal studies in arithmetic, and it is safe to assume that they had already acquired the basic schemas for solving word problems in arithmetic.

The results of the study seem contrary to results of other studies that have found a strong influence of language on the comprehension or understanding of word problems. However, we should bear in mind that the comprehension and understanding of the textual propositions of word problems is truly a thoroughly linguistic process. This process of constructing the propositional text base of the problem is an important component of the problem-solving process (Kintsch & Greeno, 1985). As such, problems in this component could lead to failures in the problem-solving process, as shown in the studies cited earlier in this article. But it is a process that does not require the application or use of more mathematically abstract conceptual and procedural knowledge.

Therefore, our understanding of the role of language in word problem solving among bilinguals cannot be captured in a simplistic proposition about the relationship between language and problem solving. Instead, the role of language in the word problem-solving process is defined in relation to the specific components of the process. There are components of the process that are more closely tied to linguistic processes, and, thus, linguistic factors are likely to have an influence on the successful execution of the process. As such, characteristics of the bilingual problem solver's language skills (e.g., relative proficiency in first and second languages) are likely to affect these processes.

However, there are components of the word problem-solving process that are mathematically abstract in character, and thus, linguistic factors are less likely to have an influence on these processes. Previous research suggests that even the mathematically abstract processes in word problem solving may be affected by concrete and superficial features of the problems. For example, results of studies by Bernardo and Okagaki (1994) and Reeves and Weisberg (1994), among others, have shown that concrete features of the problems, such as the superficial elements of the story problems that are irrelevant to the mathematical problem structure, influence the mathematical problem solving process.

Moreover, results of studies by Bernardo (1994) and Blessing and Ross (1996) showed that these concrete features also figure in the transfer of information on mathematical solutions between analogous problems and the development of abstract problem-type schemas. But the results of the present study clearly indicate that the language of the problem is a superficial feature that is not likely to influence the process of modeling the problem structure of word problems in arithmetic.

The results of the present study revealed an unexpected difference in performance between the bilingual students whose first language was Filipino and those whose first language was English. In all dependent measures, the English-as-first-language students outperformed their counterparts whose first language was Filipino. This result might indicate an artifact of sampling, in which the English-as-first-language students may have been better students in general. There is no way of verifying this sampling artifact, as no achievement or ability data about the students were obtained on other variables.

The results might also suggest yet another effect of first language on mathematical performance. In particular, the results may be interpreted as indicating the English students learned mathematics problem solving better in the English-language mathematics classroom than did the Filipino students. In the Philippines, education policy requires that English be used as the medium of instruction in mathematics (and a few other subjects). Thus, in all Philippine classrooms, teachers lecture and discuss mathematical concepts and procedures in English, using English-language problems, materials, and resources, and requiring students to discuss and respond in English.

It is plausible to argue that students whose first language was English were more successful in engaging this learning environment, but there is no way to definitely verify this interpretation based on the current data. Future researchers may have to address the two alternative interpretations of this unexpected finding, but for purposes of the present study, it should be noted that students were equally likely to succeed in the problem-completion and problem-solving tasks in the two languages. Thus, the unexpected result does not substantially alter the main finding of the study.

With globalization, more people will be speaking two or three languages by choice or by force of circumstance. Bilingualism may eventually become the normal linguistic state. In this context, psychological research on cognitive processes of bilinguals will be critical in helping us understand how bilinguals think in various domains of life, such as formal schooling. Much of the research on bilinguals has focused on how conceptual knowledge is represented in the bilingual's mind. There are fewer studies on how bilinguals engage various cognitive processes that may or may not be related to the linguistic processes.

The present study's results advance our understanding of how bilinguals engage in tasks in the domain of school mathematics. The relationship between language and the various component processes of word problem solving, by specifying that the effect of language probably depends on the nature of the specific component processes, is clarified. Thus, although language may be an important factor that shapes the cognitive processes of bilinguals, we should not stretch the importance of this factor in bilingual cognition. As the present results suggest, there are cognitive processes that may have a mathematically abstract (i.e., computational or symbolic) language of their own that may be unaffected by the languages of the bilingual individual.

REFERENCES

Bernardo, A. B. I. (1994). Problem-specific information and the development of problem-type schemata. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 20,* 379–395.

Bernardo, A. B. I. (1996). Task specificity in the use of words in mathematics: Evidence from bilingual problem solvers. *International Journal of Psychology, 31,* 13–27.

Bernardo, A. B. I. (1998). Language format and analogical transfer among bilingual problem solvers in the Philippines. *International Journal of Psychology, 33,* 33–44.

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19,* 149–163.

Bernardo, A. B. I. (2001). Principle explanation and strategic schema abstraction in problem solving. *Memory & Cognition, 29,* 627–633.

Bernardo, A. B. I. (2002). Language and mathematical problem solving among bilinguals. *The Journal of Psychology, 136,* 283–297.

Bernardo, A. B. I., & Calleja, M. O. (2005). The effects of stating problems in bilingual students' first and second languages on solving mathematical word problems. *The Journal of Genetic Psychology, 116,* 117–128.

Bernardo, A. B. I., & Okagaki, L. (1994). The roles of symbolic knowledge and problem-information context in solving word problems. *Journal of Educational Psychology, 86,* 212–220.

Blessing, S. B., & Ross, B. H. (1996). Content effects in problem categorization and problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22,* 792–810.

Carpenter, T. P., Moser, J. M., & Bebout, H. (1988). Representation of addition and subtraction word problems. *Journal for Research in Mathematics Education, 19,* 345–357.

Clarkson, P. C. (1991). Mathematics in multilingual society. In K. Durkin & B. Shire (Eds.), *Language in mathematics education* (pp. 237–246). Buckingham, England: Open University Press.

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology, 20,* 405–438.

De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education, 18,* 460–470.

De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representation and solutions. *Journal of Educational Psychology, 82,* 359–365.

Francis, W. S. (1999). Cognitive integration of language and memory in bilinguals: Semantic representation. *Psychological Bulletin, 125,* 193–222.

French, R. M., & Jacquet, M. (2004). Understanding bilingual memory: Models and data. *Trends in Cognitive Sciences, 8*(2), 87–93.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92,* 109–129.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children.* Chicago: University of Chicago Press.

Magiste, E. (1985). Development of intra- and interlingual interference in bilinguals. *Journal of Psycholinguistic Research, 14,* 137–154.

Paivio, A. (1991). Mental representation in bilinguals. In A. G. Reynolds (Ed.), *Bilingualism, multiculturalism, and second language learning* (pp. 113–126). Hillsdale, NJ: Erlbaum.

Paivio, A., Clark, J. M., & Lambert, W. E. (1988). Bilingual dual-coding theory and semantic repetition effects on recall. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 14,* 163–172.

Reeves, L. M., & Weisberg, R. W. (1994). The role of content and abstract information in analogical transfer. *Psychological Bulletin, 115,* 381–400.

Reyes, M. L. (2000). Differential effects of mathematical ability and language use on computational and conceptual knowledge of descriptive statistics. *The Asia-Pacific Education Researcher, 9,* 83–115.

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction, 5,* 49–101.

Teubal, E., & Nesher, P. (1991). Order of mention vs. order of events as determining factors in additive word problems: A developmental approach. In K. Durkin & B. Shire (Eds.), *Language in mathematical education* (pp. 131–139). Buckingham, England: Open University Press.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, The Netherlands: Swets & Zeitlinger.

APPENDIX

Complete List of Word Problems in English

Combine problems

1. Judy has 3 candies. Carlo has 5 candies. [How many candies do they have altogether?]*

2. Judy and Carlo have some candies altogether. Judy has 2 candies. Carlo has 4 candies. [How many candies do they have altogether?]
3. Judy has 4 candies. Carlo has some candies. They have 7 candies altogether. [How many candies does Carlo have?]
4. Judy has some candies. Carlo has 6 candies. They have 9 candies altogether. [How many candies does Judy have?]
5. Judy and Carlo have 8 candies altogether. Judy has 7 candies. [How many candies does Carlo have?]
6. Judy and Carlo have 4 candies altogether. Judy has some candies. Carlo has 3 candies. [How many does Judy have?]

Change problems

1. Judy has 3 candies. Then Carlo gave her 5 candies. [How many candies does Judy have now?]
2. Judy has 6 candies. Then she gave 4 candies to Carlo. [How many candies does Judy have now?]
3. Judy has 2 candies. Then Carlo gave her some candies. Now Judy has 9 candies. [How many candies did Carlo give to her?]
4. Judy has 8 candies. Then she gave some to Carlo. Now Judy has 3 candies. [How many candies did she give to Carlo?]
5. Judy has some candies. Then Carlo gave her 3 candies. Now Judy has 5 candies. [How many candies did Judy have in the beginning?]
6. Judy has some candies. Then she gave 2 candies to Carlo. Now Judy has 6 candies. [How many candies did she have in the beginning?]

Compare problems

1. Judy has 5 candies. Carlo has 8 candies. [How many candies do they have altogether?]** or [How many candies does Carlo have more than Judy?]
2. Judy has 6 candies. Carlo has 2 candies. [How many candies do they have altogether?]** or [How many candies does Carlo have less than Judy?]
3. Judy has 3 candies. Carlo has 4 candies more than Judy. [How many candies does Carlo have?]
4. Judy has 5 candies. Carlo has 3 candies less than Judy. [How many candies does Carlo have?]
5. Judy has 9 candies. She has 4 candies more than Carlo. [How many candies does Carlo have?]
6. Judy has 4 candies. She had 3 candies less than Carlo. [How many candies does Carlo have?]

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*The questions in brackets were omitted.
**These problems were also classified as combine problems because it is possible to complete the problem using a combine question.

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